

MASTERS

FINANCE

MASTER'S FINAL WORK

DISSERTATION

PENSION FUNDS: ASSET LIABILITY MANAGEMENT

LIANE COSTA GABRIEL

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ORIENTATION:

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Abstract

The financing level of pension funds and the risk of default is an issue with increasing relevance, due to the difficulties they are facing over the last years mainly resulting from changes in the demographic conditions, like the aging of the population and increasing longevity, and the 2008 financial crisis, the Great Recession. A way of optimizing the assets and liabilities and at the same time handling the risks of a fund is using Asset Liability Management models, with the best model for a fund depending on its specific characteristics and risk-return profile. This thesis will be mainly a theoretical study, where first a literature review will be done on pension plans and pension funds as well as on the importance of ALM. Then will be presented an analysis of the evolution of this risk management instrument and a description of the selected models. In the end, will be performed an application to a pension fund.

Keywords: Pension Funds, Pension Plans, Asset Liability Management, Risk Management, Funding Ratio

Resumo

O nível de financiamento e o risco de insolvência dos fundos de pensão são temas cada vez mais relevantes devido às dificuldades sentidas nos últimos anos resultantes das mudanças demográficas, como o envelhecimento da população e o aumento da longevidade, e da crise financeira de 2008, a Grande Recessão. Uma forma de otimizar os ativos e os passivos e ao mesmo tempo gerir os riscos de um fundo é usando modelos de gestão de ativos-passivos. A escolha do modelo de otimização deve ter em conta as características específicas e o objetivo risco-retorno do fundo. Esta tese é principalmente um estudo teórico, onde primeiro será feita uma revisão da literatura sobre planos e fundos de pensão e a importância dos modelos de gestão de ativos-passivos. Depois será feita uma análise da evolução deste instrumento de gestão de risco e uma descrição dos modelos escolhidos. Por fim, será feita uma análise de um fundo de pensão específico.

Palavras-chave: Fundos de Pensão, Plano de Pensão, Gestão de Ativos-Passivos, Gestão de Risco, Nível de Financiamento

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List of Abbreviations

ALM – Asset Liability Management

ASF – Autoridade de Supervisão de Seguros e Fundos de Pensões

BdP – Banco de Portugal

DB – Defined Benefit

DC – Defined Contribution

EU – European Union

FTSE 100 - The Financial Times Stock Exchange 100 Index

GDP – Gross Domestic Product

OECD - Organisation for Economic Co-Operation and Development

1. Introduction

Pension funds have an important role in the economy, being one of the largest institutional investors. The assets value of the pension funds in Portugal by the end of 2017 ascended to 19.700 million euros¹, what represents an increase of 6,6% over the previous year.

Over the last years, the demographic problems, the increasing longevity, the decline of fertility, poor stock markets' returns and falling interest rates jeopardized the financial sustainability of pension schemes. Considering, the dimension of the pension funds and the risks that need to be managed, there is an increasing concern for the development of new approaches to manage the risks and maximize the returns, also known as Asset Liability Management (ALM) models.

The decisions related to the management of a pension fund, the definition of strategies and policies are most of the times based in ALM models. Depending on the objectives, there are several kinds of models, that have gone through an evolution process and the right model for a pension fund will depend on its specific characteristics and complexity of the problem.

In this thesis, first will be done a review of the relevant literature about pension funds and Asset Liability Management and why this risk management approach is so important for the financial sustainability of a pension fund. Afterwards, the evolution of ALM models over the years will be presented and selected models will be described. And in the end, it will be performed an analysis of the BdP defined benefit pension plan under the principles of ALM.

¹[https://www.asf.com.pt/ISP/Estatisticas/fundos/estatisticas_anuais/historico/Montantes%20geridos%20\(2017\).pdf](https://www.asf.com.pt/ISP/Estatisticas/fundos/estatisticas_anuais/historico/Montantes%20geridos%20(2017).pdf)

2. Pension Funds

Pension systems should have three pillars, guaranteeing that after retirement a person will be able to keep enough income to have the same life standard. The first one is the mandatory public pension, the second is the voluntary occupational pension plans, that are usually the responsibility of the employers, and the last one is the personal savings, comprising the savings that each one makes while working for retirement time (Garcia, 2013).

The focus of this thesis are the occupational pension funds and there are some important concepts to address. First, a pension fund is a fund that joins the contributions from pension plans set up by employers, unions or other organizations and its objective is the control of the assets to provide retirement benefits. A pension plan is a scheme that establishes the rules to access early or age retirement and disability or death benefits, where the benefits can be a lump sum, an annuity or a conjugation of them. The sponsor is a legal person whose pension plans are funded through a pension fund, managed by a specialized institution, that can be a pension's fund manager company or a life insurance company (Garcia, 2014a, 2014b).

Regarding the classification of a pension fund, it can be called a public pension fund when regulated by the public sector law and the government is responsible to guarantee the payment of the benefits. By the other side, it is called a private pension fund if regulated by the private sector law and the private sponsors are responsible to guarantee the payment of the benefits.

A pension fund can also be considered closed when there is only one sponsor or having more than one, they are all part of a business association or have a professional or social connection. It is open when the sponsors do not need to have any relationship and the decision of the inclusion of new sponsors only depends of the managing company of the fund.

The private pension plans can be classified as a defined benefit plan or as a defined contribution plan. The main differences between them are that the defined benefit plan is independent of the contributions and of the financial conditions, and the benefits are set in advance, with the plan sponsor bearing the investment risk. While in a defined contribution plan the benefits remain uncertain, depending on the development of investments' value and contributions. Figure A.1, presented in the appendix, has the percentage of pension's assets by type of pension plan and the differences across countries are visible. Portugal, from the selected countries, is among the ones with higher percentage of pension's assets in defined benefit plans (71.3% in 2016). For several years, most of the pension plans were defined benefit, but nowadays, with the difficulties that the sponsors have in assuring the payment of the benefits, we are assisting to a transformation and they are now becoming defined contribution plans. Sweeting (2007) surveyed FTSE100 companies and concluded that the type of pension plan gives important information about a company, with differences across industries, e.g. Information Technology firms more likely will have a defined contribution plan.

A pension plan can be classified as contributive if the plan is financed by both the employee and employer, and non-contributive if there is no sharing of financing responsibilities and the employer is the only responsible for the payment of the contributions.

For a company some of the advantages of having a pension plan are the increase in productivity and well-being of the staff and the turnover of the oldest employees is facilitated by assuring they can afford to retire. It also improves competition for the best employees, being a way of retaining and attract the best ones. Additional reasons might include the pressure made by trade unions and employee's commissions, tax incentives comparing with other work remunerations and it can be considered as a part of the social responsibility of the company. While for the workers, with the uncertainty of the social security systems, having an income when retired from the private sector is a way of assuring an income in the retirement time.

Concluding, a plan design will be an output of the weight given by the sponsor to each of these motivations.

Some of the sources of risk for a pension fund are the actuarial risk, the longevity risk, the risk of default and the indexation of the value of the pension funds. The indexing benefit payments is the increasing of the nominal benefit payments in line with inflation to assure that the purchasing power of retired people is not affected for increases in prices and wages. Fluctuations in currency, volatility, the default of the assets and uncertainty of the financial markets' development are other major sources of risk that need to be managed (Drijver, 2005).

Mainly due to the increasing life expectancy and the aging of the population, one of the actions that mostly of the OECD countries are adopting is the increase of the statutory pension age. In the beginning of 2018, the pension age in Portugal for the social security system has become 66 years and 4 months. The Figure A.2 in the appendices has the current retirement ages for the European countries.

Figure A.3, also presented in the appendix, shows the pension funds' assets as a % of GDP for the 17 countries of the Euro Area in 2016, and from that values it is possible to assess the importance of these funds in the economies. In the specific case of Portugal, the value of the pension funds' assets is about 10% of the GDP, being worthwhile to highlight the case of Netherlands (180%), followed by Finland (51%), Malta and Ireland (both with 39%).

The financial distress that some pension funds faced this century, increased the importance of the regulations in this area. In Portugal the entity that regulates the pension funds is ASF. Additionally, the Banco de Portugal Pension Fund Management Society also defines some internal more restrictive regulations. For example, in the end of 2017 the minimum solvency ratio was 53,2% (ASF regulation) and the minimum funding ratio was 98,1% (BdP

regulation). Regarding the solvability of a pension fund, the funding ratio is a key concept that can be defined as the portion of the actuarial liability that is funded by the fund value.

3. ALM for pension funds

Asset Liability Management (ALM) is a risk management approach, which considers the assets, the liabilities and the regulations from the supervisors in the exercise of matching assets and liabilities, assuring that over the time the fund will maintain a certain level of solvency. The 20th century demographic problems and the financial crisis of a recent past contributed for the increasing importance of Asset Liability Management techniques in the economy (Toukourou and Dufresne, 2015).

Looking at Figure A.4 in appendices the dimension of the aging of the population problem is observable. For example, in Portugal the old-age dependency ratio was 19.6% in 1975, 34.6% in 2015 and in 2050 it's expected to be 73.2%, what means that in 35 years the number of people older than 65 years per 100 people of working age (20-64) will more than duplicate. This tendency is present in Portugal as well as in the other countries, anticipating the problems that the pension systems will face and the need for efficient risk management tools.

ALM models are an essential instrument for a pension fund to meet their obligation and to assure a certain funding ratio, that has a minimum value defined by the supervisor. Bad modeling or even the absence of modeling of the ALM in some cases may be the explanation for the pension funding crisis. (Berardinelli et al., 2007).

ALM management should follow some principles, basically identify the risks and the potential impact in the fund value and learn how to manage them efficiently, with a set of procedures well defined and assure that the investment structure is adjusted to the risks. ALM decisions not only have to take in consideration the different objectives as well as the restrictions imposed by the supervisor. Another important point is that a fund that wants to maximize the returns without giving a special attention to the potential loss will have a different

ALM model than one that wants mainly to neutralize the losses. So, each case is a case and a careful evaluation of the objectives, restrictions and conditions should be completed to fit the right ALM model and to select the right investment portfolio having in mind the risk-return maximization (Boender, Dert, Heemskerr and Hoek, 2007).

The results of the decisions made by the board of a pension fund directly affect the different involved parties. There are the active participants that, if the fund is contributive, are worried with changes in the contribution rate, and the part of these members that are almost in the retirement age also have interest in the level of indexation of the pension. The retired persons and surviving relatives are mainly affected by indexation level of the benefits they receive, to be able to face the inflation changes over the years. Another important part is the sponsor that pays contributions and is also the part with a higher sensitivity to financial progression of a fund, assuming some of the risks. For example, if the funding ratio is below a certain level, the sponsor may have to restore the value. However, it is also important to refer that he may also benefit if the fund is prospering. The sponsor is also interested in the administrative costs. For last, there is the supervisor, to whom the pension fund board members must report and justify their decisions. It may be difficult to conciliate the interests of all these parties, once a decision that satisfies one may have conflicts with the best interest of another, this can make the ALM a challenging process (Cannas, 2010).

Some important concepts are the solvency of the pension fund, which is the ability of a pension fund to assure all the payments in the long-run at a certain moment it is valued, as well as the funding ratio, that is the amount of assets over liabilities. It must be guaranteed with a large probability that the solvency restrictions are accomplished as well as the returns are maximized. Consequently, a concern of the board of a fund is the risk of underfunding, case in which the value of liabilities is higher than the value of the assets. As it can be seen in Figure A.5 in appendices, the average funding ratio of occupational defined benefits pension plans in

Portugal was 106% in 2012 and 103% in 2016, which means that in average the assets are covering the liabilities of the plans. It's also observable that in the selected OECD countries, in general, there aren't significant differences in the funding ratio from 2012 to 2016.

The valuation methodology of assets and liabilities is a key issue to assess the financial position of a pension fund and to compare its performance over the time. For different funds to be comparable at a certain time an evaluation of the impact of different accounting standards should be performed.

The Figure A.6 presented in the appendix shows the allocation of private pensions by assets type in Portugal for 2016. Bills and bonds are the more representative, being 56% of the assets, followed by 19% in shares, 18% in cash and deposits and for last 7% in others.

When applying an ALM model to a pension fund and choosing the investment portfolio, the risk-efficiency and well-diversified portfolios should be the basic concepts to apply. Considering that the efficiency of a fund is dependent of the portfolio selection, the buffer capital and target funding ratio. An ALM problem has as a key component the identification of the features with impact in the risks and returns, including the quantification of the effect of each possible decision, based in different risk return measures. According to Markowitz (1952), a decision is more efficient than others if it results in a higher expected return for the same level of risk or in lower risk at the same level of expected return.

ALM includes the development of future evolution scenarios for pension fund assets and liabilities, given certain assumptions about economic, financial and biometric variables (Blome, S. et al. 2007).

A pension board has a wide variety of instruments to manage the funding ratio, always taking in consideration the interests of the involved parties, namely pension policy, pension system, indexing policy, reinsurance policy, investment policy and finally contribution policy.

With effects in the liability side, the pension policy comprises the different decisions with respect to the pension fund, pension system are the rules with respect to the payment of the benefits, indexing policy is a key factor in the valuation of liabilities and future benefit payments, and finally the reinsurance policy is related with the possibility of a pension fund to sell part of the risks to an insurance company. Affecting the assets valuation, the contribution policy is connected to the possibility of changing the contribution rate and the investment policy is the decision of the asset classes in which the fund should be invested (Cannas, 2010).

4. ALM models for pension funds

In this section, it will be presented a brief description of some Asset Liability Management models that are applied to achieve an optimal asset allocation for a certain level of risk-return pretended and their evolution over the years.

4.1. The evolution of ALM models

The ALM for pension funds has its origins in the 19th century. In that time the insurance companies were responsible to manage the pension funds, it was before the sponsors to have autonomy to lead the investment decisions. The funds were invested in fixed-income products and the objective was the cash-flows matching of assets with liabilities. According to Seburn (1991), this methodology was used from 70s of the 19th century until the 60s of the 20th century.

The ALM modelling for pension funds, while recognized quasi-independent investments, had its beginning with the Dedication model. Leibowitz was the responsible for this approach, where the matching of assets and liabilities remains to be the objective and the name of the model comes by the logic of each cash inflow being dedicated to pay a specific cash outflow. Another key point was to invest the entire portfolio on bonds, assuring, at the same time the needed cash flows over the problem time horizon. The advantages of the Dedication model are the reduction of risks, no problems with decisions of asset allocation, passive management, the objective is well-known (matching of cash-flows) and the cash flows are easily predictable. On the other hand, the disadvantages passed through the complex mathematical models, the difficulties of construction, the need for accurate projected liability benefit payments and future values to match with all the uncertainties. Additional drawbacks were the reduction of the active bond managers role and pension consultants specialized in asset allocation and high sensitivity to interest rates (Leibowitz, 1986).

In the 80s, the Dedication model was replaced by the Immunization model, which consists in match the interest rate movement of liabilities in present value. It applied the principle of Macaulay (1938) as objective, minimize the volatility of the surplus by having the same duration for assets and liabilities. If the duration of the bonds held in a portfolio were matched to the duration of liabilities those bonds would fund, consequently the effects of interest rate changes could be completely mitigated and from this comes the name immunized. This approach was ignored until the long trend of increase in the interest rates, when the financial sector was forced to paying attention to the advantages of this model. Knowing that the high interest rates would lead to lock high interest rates, pension fund managers incorporated the dedication and then immunization. (Fong and Vasicek's, 1984).

Until the early 80s, the use of these models in ALM resulted well, but then the interest rates began to fall and the risks of using dedication and immunization models have become too high. As a solution, Leibowitz and Weinberger (1982) adjusted these models to the Contingent Immunization, that can be seen as a form of portfolio insurance. In this approach the portfolio stays in an active management mode while the portfolio asset value is above a certain level and it enters in an immunization mode when it falls below that value. The basis of these models is the cash-flow matching, an accurate discount rate is essential once it is used to compute the present value and the duration of a liability.

These earliest ALM models are commonly called deterministic, because in these approaches the future cash flows are estimated and assumed to be certain. New models were then developed, having as main difference from the preceding the stochastic future stream of benefit payments, though the optimal portfolio continued to be found by duration matching techniques. Other examples of these models are Cox et al. (1985) and Norris and Epstein (1989). Deterministic methods have demonstrated to be inefficient due to some problems, mainly related to the difficulties managing the uncertainty.

Only in the late 1980s simulation started to be used to solve ALM problems, as can be seen in Van der Meer (1989). Allowing to create a wide number of possible scenarios for the evolution of the financial position, made this method to become popular. Another important feature of simulation method is the incorporation of the long-term investment horizon of about 30 years, requiring a model to be dynamic and include the restrictions from supervisors. In the other side, an important disadvantage of the use of simulation is that a lot of parameters must be decided.

A possible way to overcome the simulation problems is the use of stochastic linear programming models, where, instead of being exogenous, the decisions are now endogenous to the model. The problem of uncertainty is incorporated by stochastic methods, mainly with the surplus optimization.

In the last decades, new models have been developed for stochastic programming, these techniques are complexes, flexible and a principal tool for the efficiency of Asset Liability Management. Consigli and Dempster (1998), Kusy and Ziemba (1986), Kouwenberg and Zenios (2006) are some examples of authors that applied multistage stochastic programs to Asset Liability Management for pension funds.

Over the last years, the ALM models became more complexes, moving from the one-period static type to multi-period dynamic models. These more sophisticated models involve the consistent stochastic simulation of assets and liabilities, running Monte Carlo simulations. This scenario analysis also is widely used in ALM to model economic risk and return factors. It simulates multiple scenarios of future development of economic variables and then assesses the impact in the objectives. The simulation scenario model starts exploring how ALM strategies behave in various scenarios taking in consideration the costs and the risks. Different risks and returns are computed according to each strategy. Afterwards, comes the phase of

evaluation and decision making. This process is continuously carried out until an optimal strategy that maximizes the risk-returns for the pension fund, sponsors and trustee is achieved. The scenario analysis is sometimes preferred over other approaches because of its flexibility, it doesn't give only the optimal solution, it's possible to explore more than one scenario and the facility that the decision makers will have interpreting the results and using them in a practical way and not just as a theoretical experiment (Angela Gallo, 2009).

The principles of scenario analysis are the incorporation of external uncertainties, for instance interest rates, equity market value, inflation and actuarial factors are used to build scenarios, that represent the possible future developments. The accuracy of the generated scenarios and parameters evolution are a key factor in modeling process and consequent decision process. The scenarios based in economic situation simulate the evolution of macroeconomic variables such as long-term interest rates and inflation. After that, financial markets variables are incorporated as, for example, yield curve, dividend yields, stock indexes and currency exchange rates.

Summarizing, there are a wide range of methods in Asset Liability Management, that can be classified in four main categories: stochastic programming, dynamic programming, portfolio theory and stochastic simulation. Before going deeper, it is important to have in mind that the process of ALM modeling can be quite challenging and most of the models are too demanding to be applied.

4.2. ALM optimization using simulation – Yu, Huang, Chen and Lin (2012)

The work developed by Yu et al. (2012) is a multi-period discrete-time asset liability optimization using a simulation model and includes an evolution strategy algorithm to create a defined contribution pension plan that can match the target liability and decrease the downside risk. The advantages of a multi-period asset allocation relatively to a single-period is to support for a longer time underfunding, what tends to improve the investment returns when considered the entire period. The authors purpose is to find an effective asset allocation to pay the pension benefits. With the results of the model, fund managers and investors can evaluate their targets and then meet their obligations. If after an investment period the market conditions have changed, the model must be estimated again to obtain the new effective asset allocation for the rest of the period. As the liabilities of a pension fund are long-term, the investment horizon must consider a long planning investment horizon.

The equation used to evaluate the assets' value of the portfolio at maturity is given by:

$$A(n) = \sum_{t=1}^n c\% \times S_t \prod_{i=t}^n \left[\sum_{j=1}^4 P_{ij} \times (1 + r_j(i)) \right]$$

where P_{ij} is the proportion of each asset j in time i ;

$A(n)$ is the value of assets in the end of the term;

$c\%$ is the contribution rate;

S_t is the salary in year t ;

$r_j(i)$ is the investment return of asset j in time i .

As only four asset classes were considered for this model, $j=1,2,3,4$.

The matching of assets and liabilities is a conservative approach and is a very sensible objective. A possible way to overcome this problem is to define the target liability - $L(n)$ - using as an index the income replacement rate and can be defined as:

$$L(n) = 80\% \times S_x \times \ddot{a}_x$$

Where S_x is the salary at age x , x is the retirement age and \ddot{a}_x is the value of an annuity of 1 per annum payable annually in advance for a life attaining age x .

The authors incorporated the investment performance during the accumulation period by setting that yearly, the target liability must increase as the larger value between 5% and the inflation rate in year t (rpi_t). So, the target liability at time t can be computed as:

$$L(t) = [L(t-1) + c\% \times S_{t-1}] \times \max\{1.05, rpi_t\}$$

With $L(0)=0$ and $t=1, \dots, n-1$.

The investment portfolio can be restructured each five years period. As a simplification if the proportions could be changed every year, it would be computationally demanding. So, considering all these factors the objective function is:

$$\text{Min } \theta \times E \left[(A(n) - L(n))^2 \right] - \sum_{k=1}^{k=\frac{n}{5}-1} E[A(T_k) - L(T_k)]$$

Where θ reflects the importance of the matching of assets and liabilities in the retirement time. The objective function incorporates the minimization of the tracking error in the terminal date and the need for a reduction in the downside risk.

The inputs of the objective function are the contribution rate and the proportion of the asset classes in the consecutive portfolios. The authors started by determining the strategic portfolio composition to subsequently simulate 4000 scenarios for the assets returns in the next 40 years. The model pretends to help defining a contribution rate and portfolio composition

overtime. The conclusion for the scenario-based simulation are that more risky investments will lead to higher returns, lower contribution rate and are at the same time responsible for the higher losses.

Afterwards, in the first years the investment in stock markets should be preferred and gradually replaced with investments in assets that have less risk until the retirement time. So, optimization model can find promising results in helping managers and investors in achieve higher returns or reducing the risks in their investment decisions.

4.3. Robust Optimization – Platanakis and Sutcliffe (2017)

Another relatively new technique in ALM for pension funds is the robust optimization. This method recognizes that the market parameters of an ALM model are stochastic but are within certain uncertainty sets. Over the last years, it has attracted attention and is recognized as a powerful and efficient technique while solving ALM problems, it was applied to portfolio optimization and asset management.

The robust optimization solves the maximin problem, that is it assumes that each stochastic parameter assumes the most unfavorable value in its uncertainty set. It has the advantage of being much easier to solve than the stochastic programming, which is computational demanding. This solution tends to eliminate extreme solutions and consequently originates investment in diversified portfolios.

The mentioned authors assume that the stochastic assets and liabilities are described by the following factor model:

$$\tilde{r}_{A,L} = \tilde{u}_{A,L} + \tilde{V}^T f + \tilde{\epsilon}_{A,L}$$

Where,

$\tilde{r}_{A,L}$ is a joint column vector with n_A+n_L elements that contains asset and liability returns;

$\tilde{u}_{A,L}$ is a joint column vector with n_A+n_L elements that contains the random asset and liability returns that drive the risky assets and liabilities;

\tilde{f} is the column vector with m (number of factors) elements contains the factor returns that drive the risky assets and liabilities;

\tilde{V} is the matrix with m rows and n_A+n_L columns containing the corresponding uncertain factor coefficients;

$\tilde{\epsilon}_{A,L}$ with n_A+n_L elements is the column vector of uncertainty disturbances.

The \tilde{V} is part of an elliptical uncertainty set denoted by S_u . The elements of $\tilde{u}_{A,L}$ and the diagonal elements of the covariance matrix of the disturbances (\tilde{D}) lie within certain intervals which are represented by the uncertainty structures S_{mean} and S_d , respectively. These parameters depend on w that represents the level of confidence.

As this is a factor model one of the advantages is that there is no need to estimate the covariance matrix of the asset-liability returns, just the covariance matrix of the factor returns.

The robust optimization problem we is given by the following maximin problem:

$$\underset{\Phi_A}{\text{maximize}} \left[\frac{\min_{\{\tilde{u}_{A,L} \in S_{mean}\}} [\tilde{u}_{A,L}^T \Phi_{A,L}]}{\sqrt{\max_{\{\tilde{V} \in S_u\}} [\Phi_{A,L}^T \tilde{V}^T F \tilde{V} \Phi_{A,L}] + \max_{\{\tilde{D} \in S_d\}} [\Phi_{A,L}^T \tilde{D} \Phi_{A,L}]}} \right]$$

subject to: $1^T \Phi_A = 1,$

$$\Phi_{A,i} \geq 0, \quad \forall i = 1, \dots, n_A,$$

$$\begin{aligned}
 - \sum_{i \in \text{class} X} \Phi_{A,i} + \Theta_{\max}^{\text{class} X} 1^T \Theta_A &\geq 0, & \forall \text{class} X, \\
 \sum_{i \in \text{class} X} \Phi_{A,i} + \Theta_{\min}^{\text{class} X} 1^T \Theta_A &\geq 0, & \forall \text{class} X
 \end{aligned}$$

Where $\phi_{A,L}$ denotes the joint column vector of asset proportions ϕ_A and liability proportions ϕ_L . $\Theta_{\min}^{\text{class} X}$ and $\Theta_{\max}^{\text{class} X}$ are the minimum and maximum values for each asset class X, with X equal to equities, bonds, real estate and others.

The objective of this function is to maximize the Sharpe Ratio under the worst circumstances, the maximin. The numerator is the worst-case mean return and the denominator is the worst-case variance, both depending on the parameter w, as this parameter is increased the size of the uncertainty set also increases.

They used three benchmarks. The first is the actual portfolios chosen by the USS fund. The second is a modified version of Sharp and Tint model, where the objective function maximizes the expected return on the asset portfolio in excess of the liability portfolio, divided by the standard deviation of these excess returns. So, it is given by:

$$\text{maximize}_{\Phi_A} \frac{\Phi_{A,L}^T \mu_{A,L}}{\sqrt{\Phi_{A,L}^T \Sigma_{A,L} \Phi_{A,L}}}$$

subject to: $1^T \Phi_A = 1,$

$$\Phi_{A,i} \geq 0, \quad \forall i = 1, \dots, n_A,$$

$$- \sum_{i \in \text{class} X} \Phi_{A,i} + \Theta_{\max}^{\text{class} X} 1^T \Theta_A \geq 0, \quad \forall \text{class} X,$$

$$\sum_{i \in \text{class} X} \Phi_{A,i} + \Theta_{\min}^{\text{class} X} 1^T \Theta_A \geq 0, \quad \forall \text{class} X,$$

where the $\Sigma_{A,L}$ is the sample covariance matrix of the assets and liabilities returns.

The last benchmark is Bayes-Stein. In this approach the estimates of the inputs for a portfolio problem are based on the knowledge that estimated returns with a higher distance from the norm have a higher chance of containing estimation errors than estimated returns close to the norm. To face these errors, the estimation of the input parameters are the weighted sum historic returns for each asset and a global estimate of returns.

The fact that the robust optimization has as objective the maximin means that will tend to have a better performance than other techniques when the market falls and the opposite happens in a rising market. In the end, an evolution of the performance of the various approaches is executed to find the best solution.

The authors found that the robust optimization outperformed the benchmarks in almost every time periods.

5. Analysis of a pension fund

In this section it will be done a brief analysis of the Banco de Portugal defined benefit pension fund, based on the Report and Accounts of the fund. It was created in 1988, it is closed, exclusively affected to the Banco the Portugal fulfillment of obligations and is divided into the pension plan and the health benefits plan.

Making a description of the fund population, by the end of 2017 the pension plan had 1054 active members and 2567 beneficiaries, while the health benefits plan had 1755 active members and 2512 beneficiaries. The integration of bank's employees who were hired after March of 2009 in the social security system and the consequent close of the pension fund to new members, have contributed for the increasing tendency of the reduction of the ratio active members/ beneficiaries over the last years. This is translated in a higher weight of the payment of the pensions in the total responsibilities, which restrains the investment policy of the fund.

In 2017, the age limit for the attribution of the pension for this fund was 65 years. In 2017, 61,2% of the liabilities were related to retired members and 38,8% to active members. The ALM of the fund generated in 2017, financial earnings of 1,5% of the liabilities, based on the position taken relatively to both assets as well as liabilities.

The Figure 5.1 has represented the evolution of the fund value and looking for the last years it's possible to conclude that in general it tends to increase over time and comparing the value of 2008 with 2017 it had a significant rise.

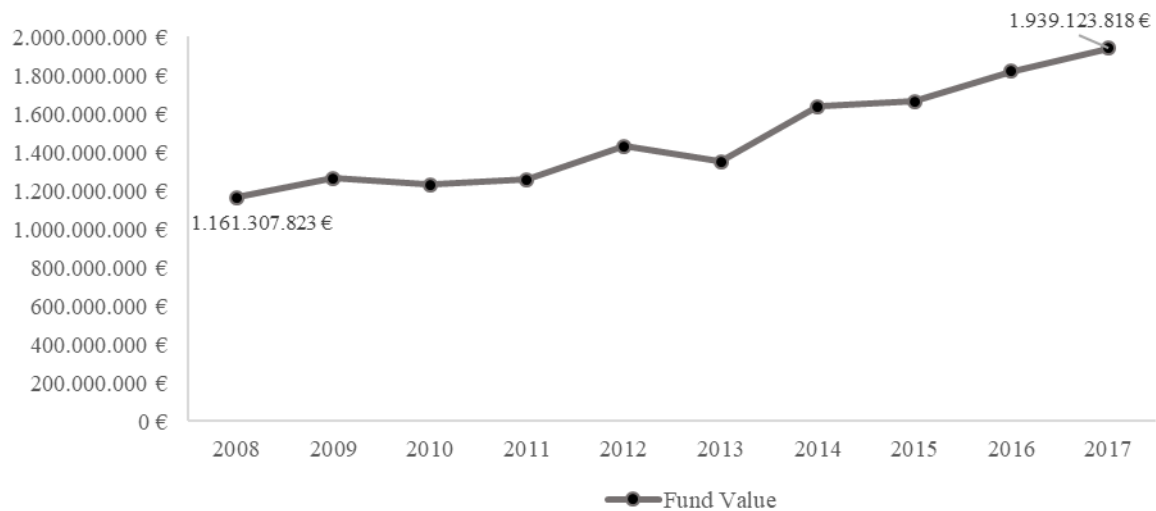


Figure 5.1 - Evolution of the BdP defined benefit pension fund value from 2008 to 2017

The evolution of the contributions and the value of pensions paid since 2008 can be observed in Figure 5.2. In one side, the value of pensions paid slightly increase over the years but without significant variations. In the other side, the value of the contributions has a high variability and, in some years for the fund to be able to meet its obligations, Banco the Portugal is called to make some additional contributions. Additionally, it's important to refer that in the specific case of this pension fund, all the contributions are made by Banco de Portugal and not by the employees.

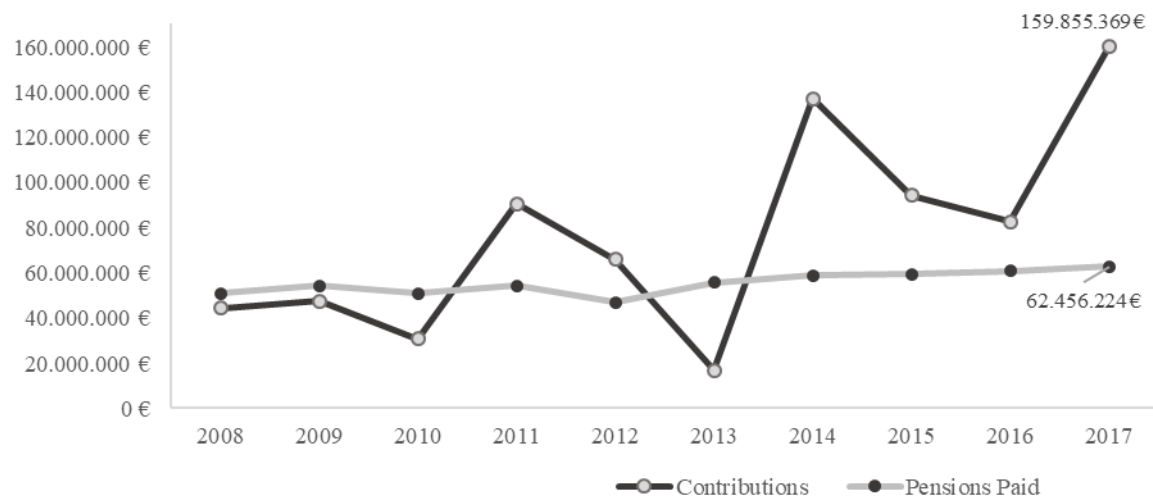


Figure 5.2 - Evolution of BdP defined benefit pension fund contributions and value of pensions paid from 2008 to 2017

Considering the total liabilities of the fund, resulting from the actuarial valuation, it's observable a significant increase over the years, as can be observed in Figure 5.3. The liabilities can be split in two parts, the responsibilities with active members and retired members (for 2017, they are 737.353.283€ and 1.160.793.477€, respectively).

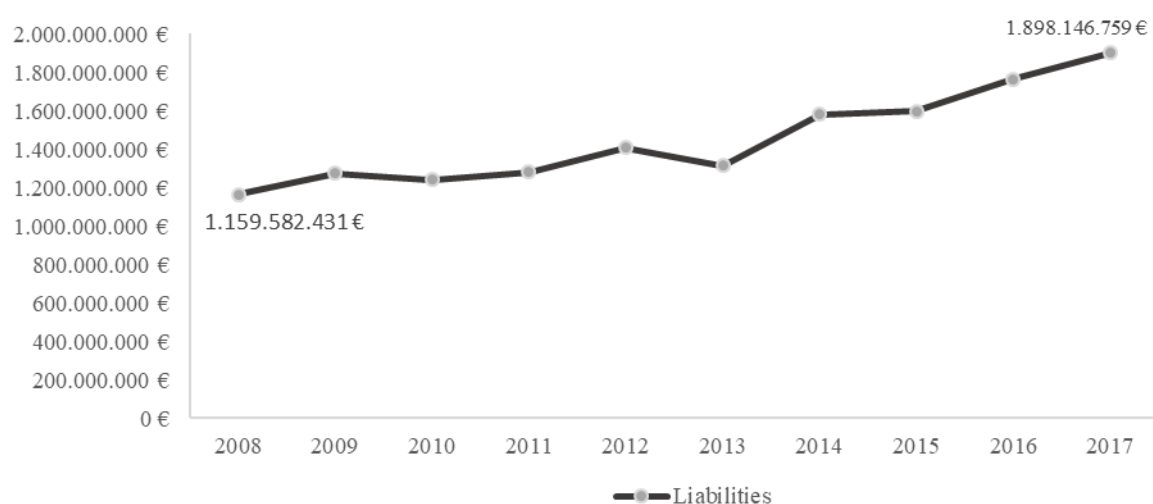


Figure 5.3 - Evolution of BdP defined benefit pension fund liabilities from 2008 to 2017

Another important fact about 2017, was the increase of 7,7% in the Liabilities mainly due to the change in the mortality assumption (+12%).

As shown in Figure 5.4 and Table 5.1, the asset allocation in the end of 2017 by the main investment products were bonds (82,9%), real estate (8,6%), stocks (7,4%) and liquidity (1,1%). As the fund has a high percentage invested in bonds and a relatively low percentage in stocks, it is possible to conclude that this fund is more concerned in minimizing the risks than maximizing the returns. Relatively to investment policy, the major part of the fund is invested on instruments of interest rates in the euro area, followed by the instruments with exposition to the real estate market, instruments with exposition to the stock market and for last the liquidity. Overlooking the fund investments, the fund over the years keeps a high exposure to the euro area countries, and the exposition to other geographic areas comes from the investment in stock markets and private debt. So, the cambial risk is very low.

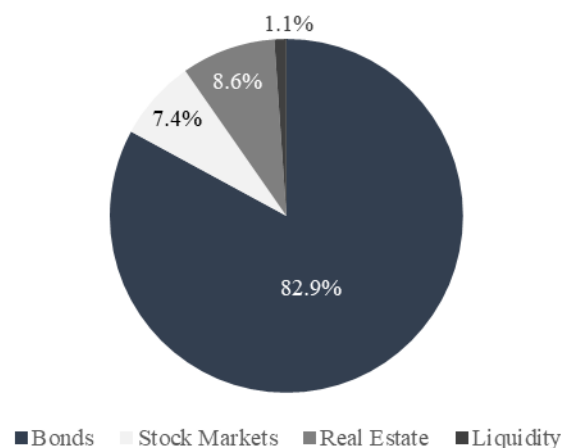


Figure 5.4 - BdP defined benefit pension fund asset allocation by asset class (2017)

Table 5.1 – BdP defined benefit pension fund investment portfolio (2017)

Bonds	82.9%
Public Debt Indexed to inflation	66.1%
Public Debt at a fixed Rate	16.5%
Public Debt	0.3%
Stock Markets	7.4%
Euro Area	2.1%
Europe Except Euro Area	1.4%
Asia-Pacific	0.4%
North America	2.9%
Emerging	0.6%
Real Estate	8.6%
Funds	1.7%
Real Estate Properties	6.9%
Liquidity	1.1%

In Figure 5.5, it's possible to see that the BdP defined benefit pension fund over the last 10 years always achieved a funding ratio above the minimum funding ratio defined by BdP regulation. The difference of the value obtained to the minimum required had the lowest value in 2011.

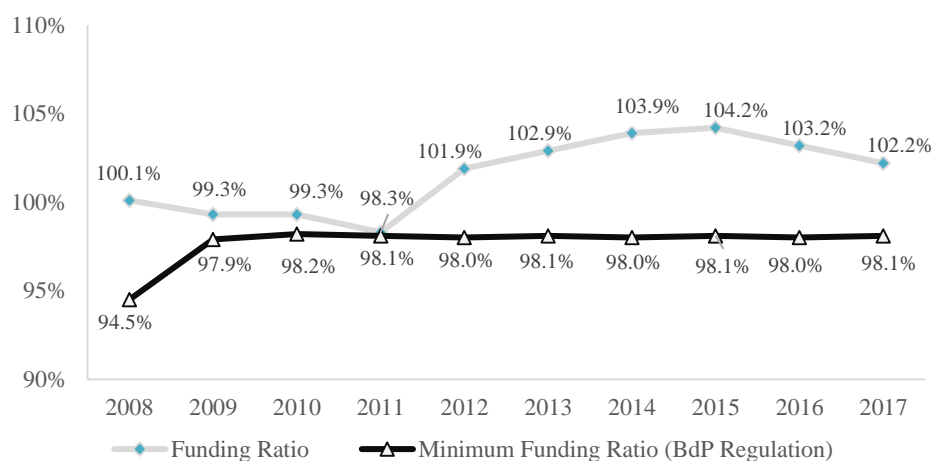


Figure 5.5 - Comparison between the minimum required funding ratio and the funding ratio of the BdP defined benefit pension fund from 2008 to 2017.

From the figure above, it's possible to conclude that the ALM strategy applied by the BdP defined benefit pension fund has been successful, and since 2012 that the assets completely covering the liabilities. The funding ratio in 2016 of this BdP defined benefit fund is in line with the average funding ratio of the occupational defined benefit in Portugal (figure 10 in appendices), both with 103%.

Although the kind of ALM model used by this fund is not public, it can be concluded from these values that the ALM model chosen probably has as objective the matching of the assets with liabilities, since the funding ratio doesn't have significant differences from 100% over the years.

6. Conclusions and Future Research

Over the years, Asset Liability Management became an essential instrument for a pension fund manager to be able to mitigate the risks and maximize the returns. As the pension funds optimization problem had increasing complexity, more sophisticated models were developed to incorporate all the factors to perform an accurate ALM exercise.

The approach used for ALM will depend if it is a defined benefit or defined contribution pension plan. For a defined benefit, the ALM objective usually is the matching of assets and liabilities, although sometimes more complex optimization exercises are performed to maximize the returns. In the other side, for defined contribution, as the benefits are not defined, the principle to apply is usually, to prefer the investment in risky assets in the first years and, with the approximation of the retirement time, replace them for less risky assets.

Regarding the two models described in this thesis, that were chosen after a review of the literature in this area, first we have the Optimization using Simulation (Yu et al., 2012) that is applied to defined contribution pension funds. In this model the benefits are not defined and the target liability for the end of the investment horizon is set based on the replacement rate. The simulation consists in running 4000 simulations for the future assets' returns, helping to define the portfolio composition and the contribution rate.

The second model studied was the Robust Optimization (Platanakis, 2017), that can be applied to defined benefit pension funds. It assumes that the market parameters are stochastic and it has as objective to maximize the worst-case scenario and find the right portfolio allocation for this purpose. The authors compared the results with other three benchmarks and the robust optimization was the one with best outcomes. This model has as main advantage to

be easier to solve than the stochastic programming and eliminates extreme solutions, what leads to a well-diversified portfolio.

The analysis of these models enables the understanding of the differences of ALM modeling for each type of pension plan. In a defined benefit the benefits are set, consequently the fund might have some difficulties in meeting its obligations and modeling will help achieve this objective. While a defined contribution fund is more flexible and the ALM problem should be faced as a way of minimizing the contribution rate and optimize the portfolio selection.

Relatively to the BdP defined benefit pension fund analysis, it is possible to conclude that the fund is using an efficient ALM strategy, considering the good results achieved over the years.

Considering the complexity of most ALM models and the limited access to essential data, it was not possible to perform an application of a model to a pension fund in this thesis. Therefore, an idea for a future work is precisely to apply an ALM model that already exists in the literature. Additionally, knowing that the need for adaptation of ALM models is constant, it would be interesting in the future the development of a model that includes all the important parameters for an accurate modelling, based on the characteristics of a specific pension fund and at the same time being computational simplified.

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A. Appendices

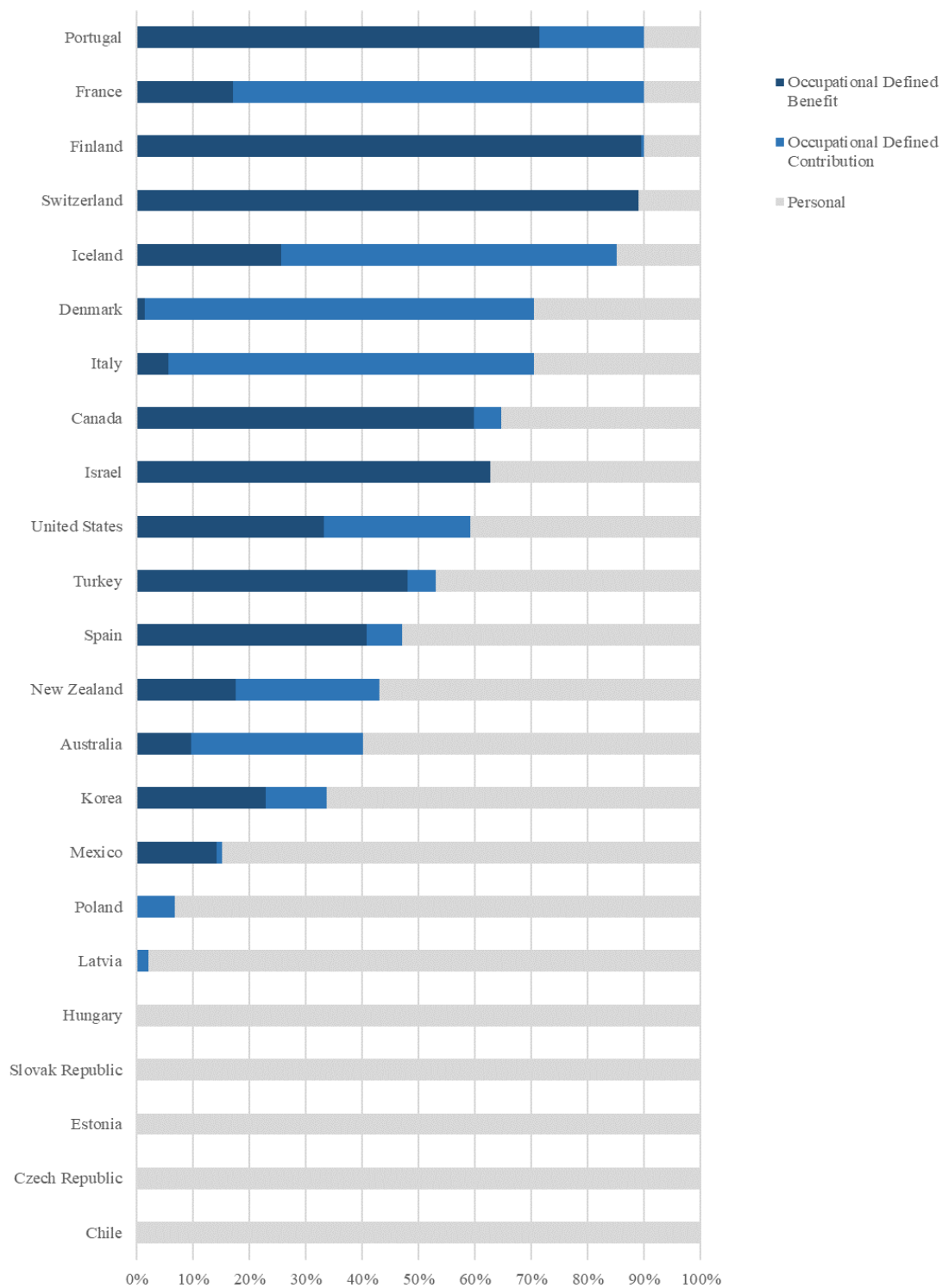


Figure A.1 - Pension's assets by type of private pension plan in the selected OECD countries (2016).

Source: OECD (2017)

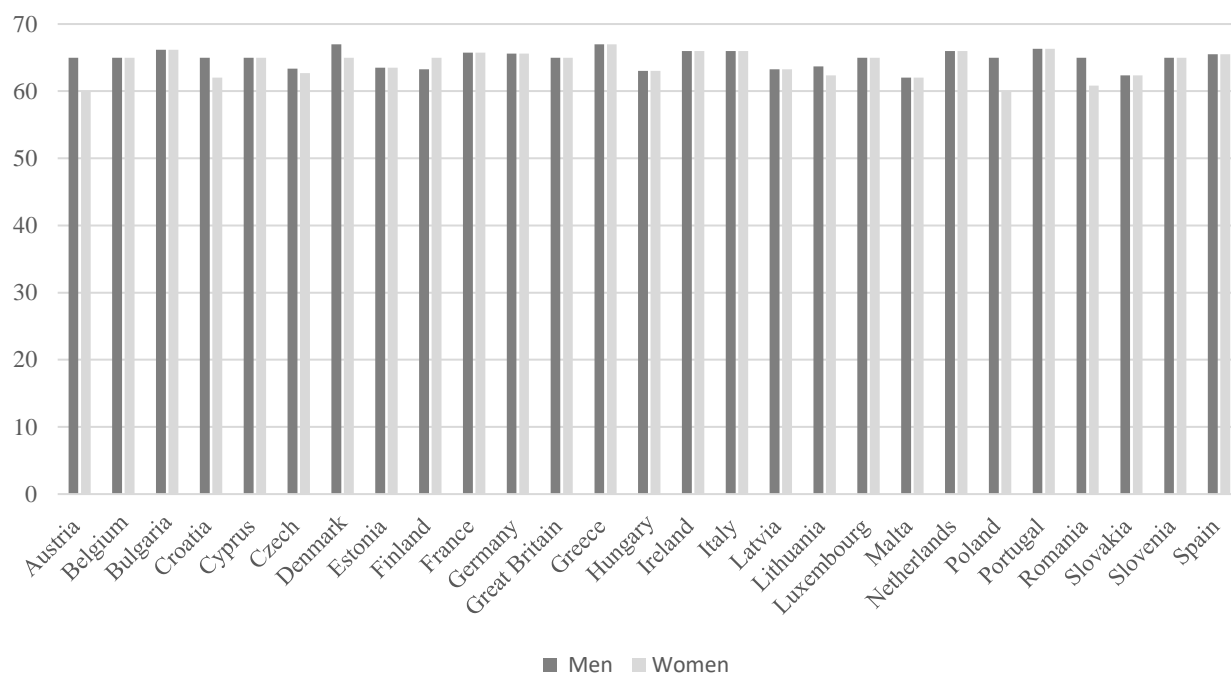


Figure A.2 - Current retirement ages (2018)²

² <https://www.etk.fi/en/the-pension-system/international-comparison/retirement-es/>

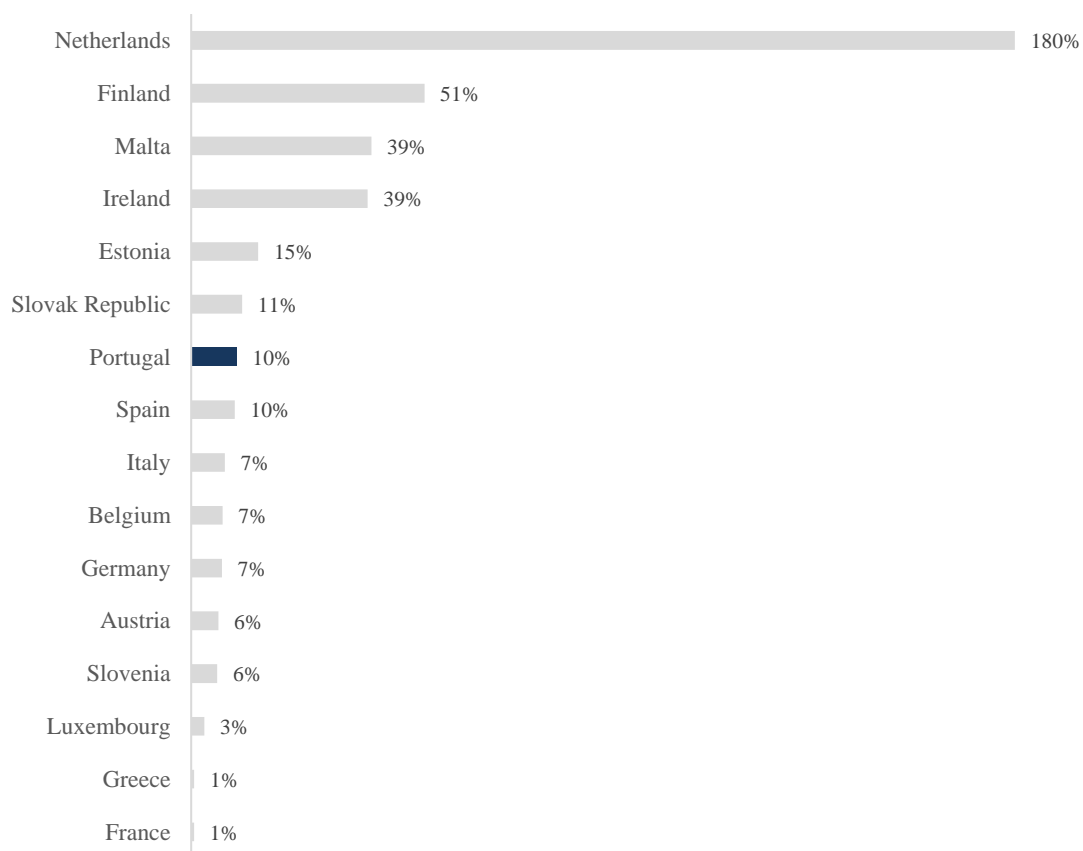


Figure A.3 - Pension funds' assets as a % of GDP in Euro Area Countries (2016). Source: OECD
(2017)

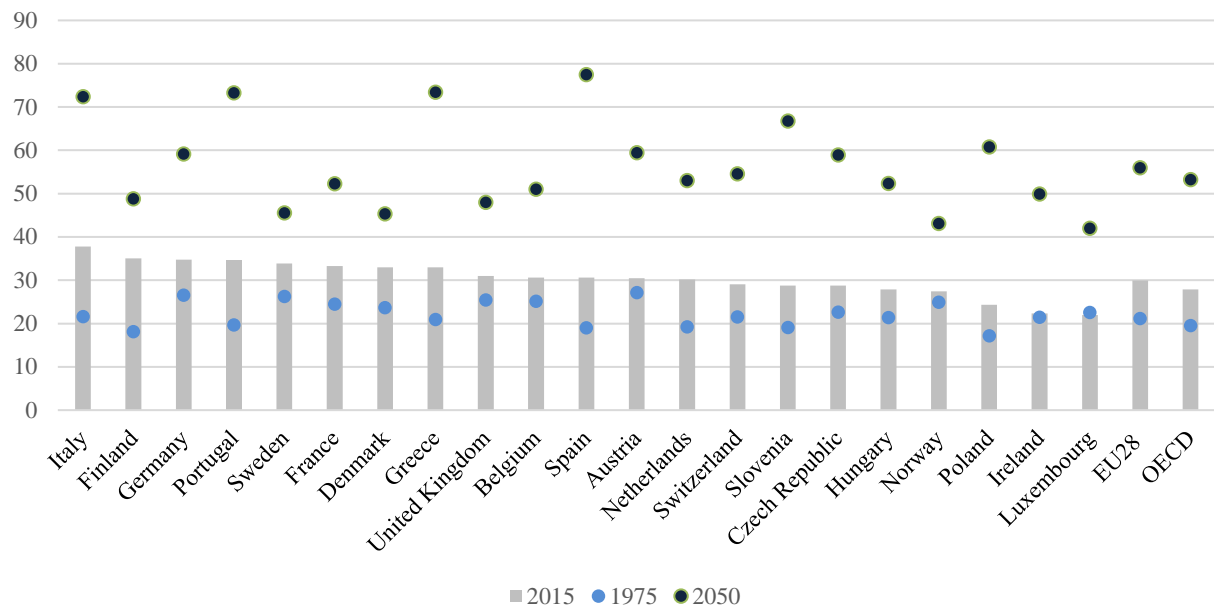


Figure A.4 - Comparison of the old-age dependency ratio (number of people older than 65 years per 100 people of working age, 20-64) for the select countries, EU28 and OECD. Source: OECD (2017)

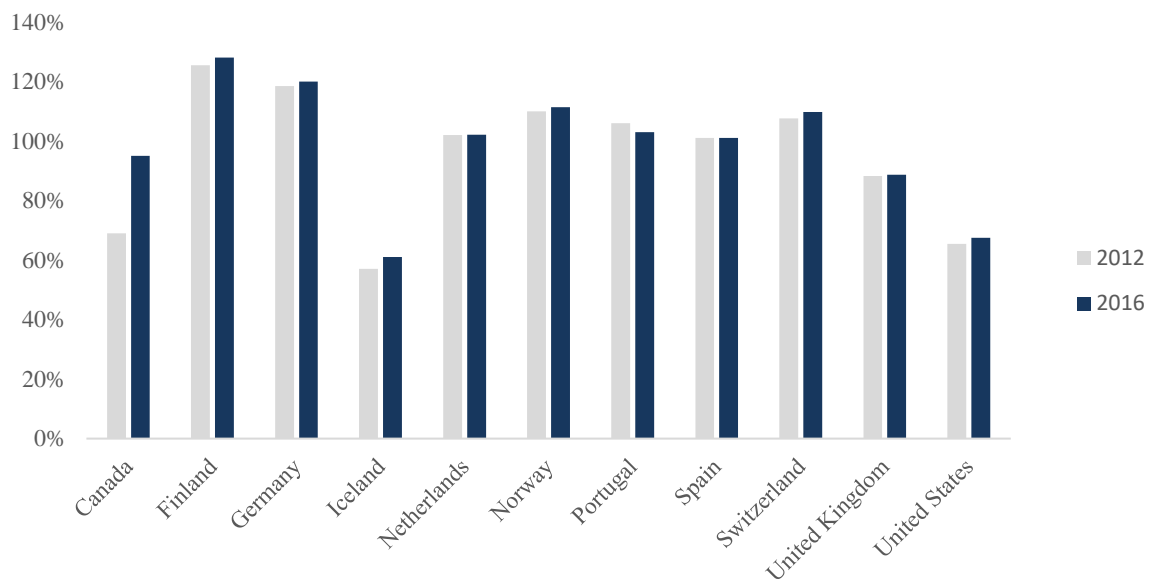


Figure A.5 - Average funding ratio of occupational defined benefits pension plans in selected OECD countries (2012 and 2016). Source: OECD (2017)

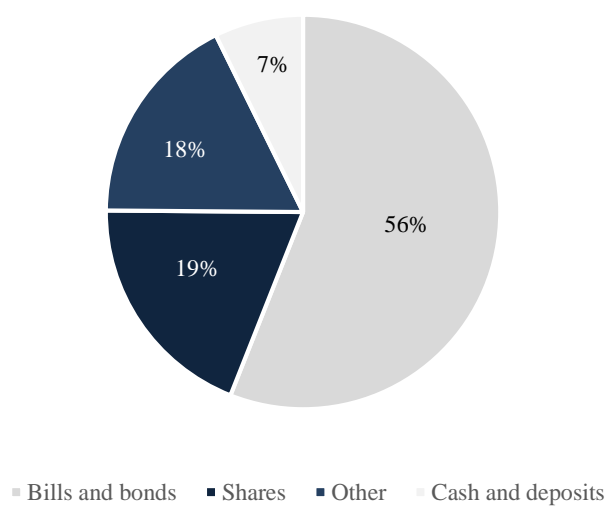


Figure A.6 - Allocation of private pension's assets in Portugal, as a percentage of total investment (2016). Source: OECD (2017)